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（答題不得寫在紅綫外）

Math 2010 E Mid term test

1. (a) Normal vector of the required plane is perpendicular to

$(3,2,4)$ and $(2,1,-3)$.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = -10\hat{i} + 17\hat{j} - \hat{k}$$

It passes through the point $(-1,1,2)$, which is on the st. line.

Since $-10(-1)+17(1)-(2) = 25$.

Equation of the plane is

$$-10x + 17y - z = 25$$

$$10x - 17y + z + 25 = 0$$

(b). Let $\gamma(t) = (t, \frac{t^2}{\sqrt{2}}, \frac{t^3}{3})$

$\gamma'(t) = (1, \frac{2t}{\sqrt{2}}, t^2)$

Arclength of the parametrized curve $= \int_0^3 |\gamma'(t)|\, dt$

$= \int_0^3 \sqrt{1+2t^2+t^4}\, dt$

$= \int_0^3 \sqrt{(1+t^2)^2}\, dt$

$= \int_0^3 1+t^2\, dt$

$= \left[t + \frac{t^3}{3}\right]_{t=0}^{3}$

$= 3 + 9 = 12$

2(a).
$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 3 & 2 & 2 & 7 \end{array}\right)$$

$$\xrightarrow{R_2-3R_1} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -4 & 5 & 1 \end{array}\right)$$

$$\xrightarrow{-\frac{1}{4}R_2} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -\frac{5}{4} & -\frac{1}{4} \end{array}\right)$$

$$\xrightarrow{R_1-2R_2} \left(\begin{array}{ccc|c} 1 & 0 & \frac{3}{2} & \frac{5}{2} \\ 0 & 1 & -\frac{5}{4} & -\frac{1}{4} \end{array}\right)$$

$\therefore$
$$x = \frac{5}{2} - \frac{3}{2}z$$
$$y = -\frac{1}{4} + \frac{5}{4}z$$
$$z = z$$

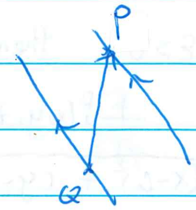
The parametric form for the line of intersection can be

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ -\frac{1}{4} \\ 0 \end{pmatrix} + z\begin{pmatrix} -\frac{3}{2} \\ \frac{5}{4} \\ 1 \end{pmatrix}$$

Direction of the line is $\begin{pmatrix} -\frac{3}{2} \\ \frac{5}{4} \\ 1 \end{pmatrix} = -\frac{1}{4}\begin{pmatrix} 6 \\ -5 \\ -4 \end{pmatrix}$

$\therefore$ Two lines are parallel.

(b).



When $z=1$, $(x,y,z) = (1,1,1)$ is a point on the line of intersection

Let $P = (1,1,1)$.

For $t=0$, $Q(1,3,2)$ is a point on the other line

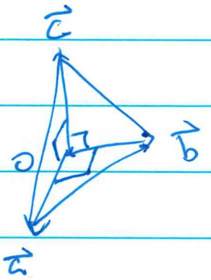
$\vec{PQ} = (0,2,1)$

The shortest distance $= \dfrac{|(0,2,1) \times (6,-5,-4)|}{|(6,-5,-4)|}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 1 \\ 6 & -5 & -4 \end{vmatrix} = \langle -3, 6, -12 \rangle$$

$\therefore$ distance $= \dfrac{\sqrt{9+36+144}}{\sqrt{36+25+16}} = \dfrac{3\sqrt{21}}{\sqrt{77}} = \dfrac{3\sqrt{3}}{\sqrt{11}}$

3.



$$H = \frac{1}{2}|(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})| = \frac{1}{2}|\vec{b}\times\vec{c} - \vec{b}\times\vec{a} - \vec{a}\times\vec{c}|$$

$$A^2+B^2+C^2 = \left(\frac{1}{2}|\vec{a}||\vec{c}|\right)^2 + \left(\frac{1}{2}|\vec{a}||\vec{b}|\right)^2 + \left(\frac{1}{2}|\vec{b}|\cdot|\vec{c}|\right)^2$$
$$= \frac{1}{4}\left(|\vec{a}|^2|\vec{c}|^2 + |\vec{a}|^2|\vec{b}|^2 + |\vec{b}|^2|\vec{c}|^2\right)$$

$$H^2 = \frac{1}{4}(\vec{b}\times\vec{c} - \vec{b}\times\vec{a} - \vec{a}\times\vec{c})\cdot(\vec{b}\times\vec{c} - \vec{b}\times\vec{a} - \vec{a}\times\vec{c})$$

Notice that for the solid right angle, $\vec{b}\times\vec{c} \parallel \vec{a}$

$\therefore$ $(\vec{b}\times\vec{c})\cdot(\vec{b}\times\vec{a}) = 0 = (\vec{b}\times\vec{c})\cdot(\vec{a}\times\vec{c})$

Similarly, $(\vec{b}\times\vec{a})\cdot(\vec{b}\times\vec{c}) = 0 = (\vec{b}\times\vec{a})\cdot(\vec{a}\times\vec{c})$

$(\vec{a}\times\vec{c})\cdot(\vec{b}\times\vec{c}) = 0 = (\vec{a}\times\vec{c})\cdot(\vec{b}\times\vec{a})$

$\therefore$ $H^2 = \frac{1}{4}\left(|\vec{b}\times\vec{c}|^2 + |\vec{b}\times\vec{a}|^2 + |\vec{a}\times\vec{c}|^2\right)$

$= \frac{1}{4}\left(|\vec{b}|^2|\vec{c}|^2 + |\vec{b}|^2|\vec{a}|^2 + |\vec{a}|^2|\vec{c}|^2\right)$

$= A^2+B^2+C^2$

4(a) For any $\varepsilon > 0$, there is some $\delta > 0$ so that

$$|f(x,y,z) - l| < \varepsilon \quad \text{if}$$

$$0 < \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} < \delta$$

(b) Consider $x = 0$, $y > 0$.

$$\lim_{y \to 0^+} \frac{\ln(0 + e^y)}{\sqrt{0 + y^2}} = \lim_{y \to 0^+} \frac{y}{y} = 1$$

Consider $x = 0$, $y < 0$.

$$\lim_{y \to 0^-} \frac{\ln(0 + e^y)}{\sqrt{0 + y^2}} = \lim_{y \to 0^-} \frac{y}{-y} = -1 \neq \lim_{y \to 0^+} \frac{\ln(0 + e^y)}{\sqrt{0 + y^2}}$$

$$\therefore \lim_{y \to 0} \frac{\ln(0 + e^y)}{\sqrt{0 + y^2}} \quad \text{DNE}$$

$$\text{and} \lim_{(x,y) \to (0,0)} \frac{\ln(x + e^y)}{\sqrt{4x^2 + y^2}} \quad \text{DNE}.$$

5.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$\therefore$ This is a plane passing through $(2, 1, 0)$
and generated by vectors $(2, 2, 4)$ and $(1, 1, -1)$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 4 \\ 1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 6 \\ 1 & 1 & -1 \end{vmatrix} = -6(\hat{i} - \hat{j})$$

$$= -6\hat{i} + 6\hat{j}$$

$\therefore$ Equation of the plane is

$$-6(x-2) + 6(y-1) = 0$$

$$x - y - 1 = 0$$

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Bonus :

$$\text{Distance} = \frac{|4\cdot 1 + 3\cdot 3 - 7\cdot 1 - 8|}{\sqrt{4^2+3^2+7^2}} = \frac{2}{\sqrt{74}}$$